ECE 344

MICROWAVE FUNDAMENTALS PART1-Lecture 4

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Terminated TL

- Input impedance , two formula
- TL repeats mag. of Voltage, mag. of Current, Z every $\lambda/2$
- EX1

  -Total voltage and current on terminated TL

  -$\mathrm{Vmax, Imax, Vmin, Imin}$

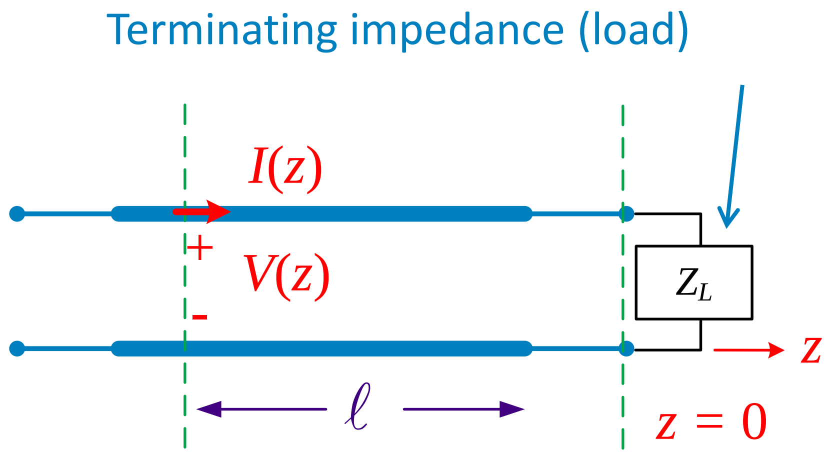
  -Reflection coefficient
- Ex2 terminated and sourced TL
- Equivalent input impedance for open circuit , short circuit and matched TL
- Equivalent TLs to lumped elements: inductors and capacitors.
- Input impedance for $\lambda/2$ and $\lambda/4$
- Quarter wavelength transformer
- Compute input impedance for TL with lumped elements or different connections.
- Voltage on TL
- SWR

Terminated Transmission Line

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

Ampl. of voltage wave propagating in positive z direction at $z = 0$.

Ampl. of voltage wave propagating in negative z direction at $z = 0$.



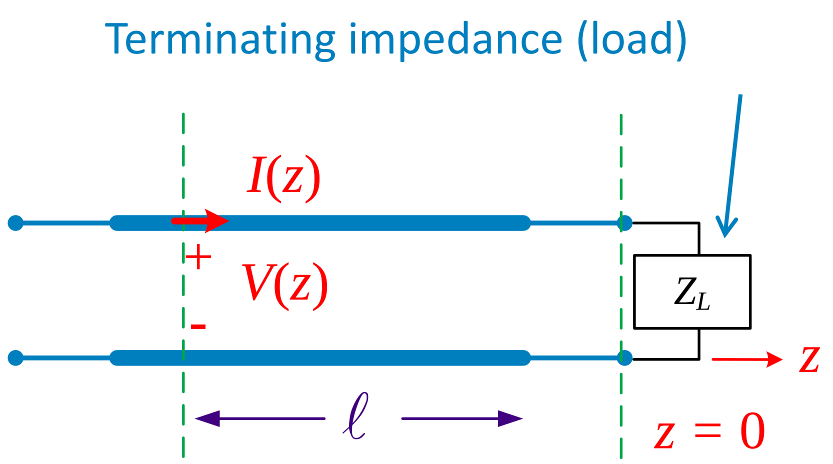
Where do we assign $z = 0$?

The usual choice is at the load.

Note: The length ℓ measures distance from the load: $\ell = -z$

Terminated Transmission Line (cont.)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$



What is $V(-\ell)$?

$$V(-\ell) = V_0^+ e^{\gamma \ell} + V_0^- e^{-\gamma \ell}$$

propagating forwards

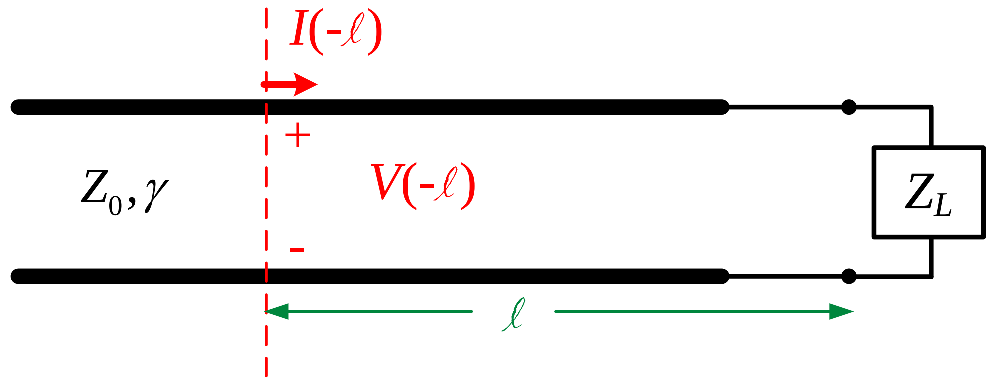
propagating backwards

The current at $z = -\ell$ is then

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma \ell} - \frac{V_0^-}{Z_0} e^{-\gamma \ell}$$

$\ell \equiv$ distance away from load

Terminated Transmission Line (cont.)



Total volt. at distance ℓ from the load

$$V(-\ell) = V_0^+ e^{\gamma\ell} + V_0^- e^{-\gamma\ell} = V_0^+ e^{\gamma\ell}\left(1 + \frac{V_0^-}{V_0^+} e^{-2\gamma\ell}\right)$$

Ampl. of volt. wave prop. towards load, at the load position (z = 0).

Ampl. of volt. wave prop. away from load, at the load position (z = 0).

$\Gamma_L \equiv$ Load reflection coefficient

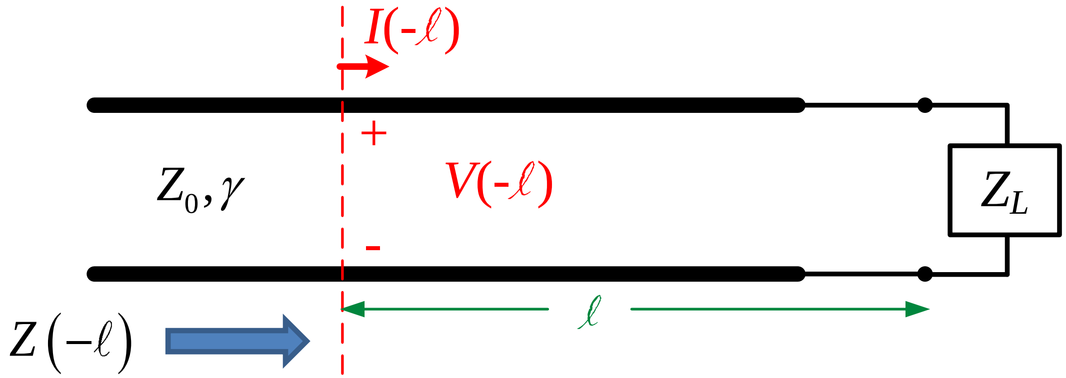
$\Gamma_\ell \equiv$ Reflection coefficient at z = - ℓ

$$= V_0^+ e^{\gamma\ell}\left(1 + \Gamma_L e^{-2\gamma\ell}\right)$$

Similarly,

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma\ell}\left(1 - \Gamma_L e^{-2\gamma\ell}\right)$$

Terminated Transmission Line (cont.)



$$V(-\ell) = V_0^+ e^{\gamma \ell} \left(1 + \Gamma_L e^{-2\gamma \ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma \ell} \left(1 - \Gamma_L e^{-2\gamma \ell}\right)$$

$$Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = Z_0 \left(\frac{1 + \Gamma_L e^{-2\gamma \ell}}{1 - \Gamma_L e^{-2\gamma \ell}}\right)$$

Input impedance seen "looking" towards load at $z = -\ell$.

Terminated Transmission Line (cont.)

At the load ($\ell = 0$):

$$Z(0) = Z_0 \left(\frac{1+\Gamma_L}{1-\Gamma_L} \right) \equiv Z_L$$

$$\Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Recall $$Z(-\ell) = Z_0 \left(\frac{1+\Gamma_L e^{-2\gamma\ell}}{1-\Gamma_L e^{-2\gamma\ell}} \right)$$

Thus,

$$Z(-\ell) = Z_0 \left(\frac{1+\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-2\gamma\ell}}{1-\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-2\gamma\ell}} \right)$$

Terminated Lossless Transmission Line

$$\gamma = \alpha + j\beta = j\beta$$

$$V(-\ell) = V_0^+ e^{j\beta\ell}\left(1 + \Gamma_L e^{-2j\beta\ell}\right)$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{j\beta\ell}\left(1 - \Gamma_L e^{-2j\beta\ell}\right)$$

$$Z(-\ell) = Z_0\left(\frac{1 + \Gamma_L e^{-2j\beta\ell}}{1 - \Gamma_L e^{-2j\beta\ell}}\right)$$

$$Z(-\ell) = Z_0\left(\frac{Z_L + jZ_0\tan(\beta\ell)}{Z_0 + jZ_L\tan(\beta\ell)}\right)$$

Impedance is periodic with period $\lambda_g/2$

tan repeats every π

$$\beta\ell = \pi$$

$$\frac{2\pi}{\lambda_g}\ell = \pi$$

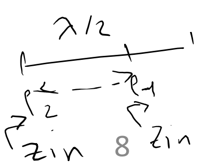
$$\Rightarrow \ell = \lambda_g / 2$$

$$\tan(\beta\ell) = \tan(\beta\ell + \pi)$$

$$\beta\ell_2 = \beta\ell_1 + \pi$$

$$\beta(\ell_2 - \ell_1) = \pi$$

$$\ell_2 - \ell_1 = \frac{\lambda}{2}$$



Example 1

USE ADS to:

-draw magnitude voltage across the line $\beta l = 2\pi$ or $\ell = \lambda$

-draw magnitude current across the line

-draw impedance across the line

observe mag(V),mag(I),Z every $\ell = \lambda/2$

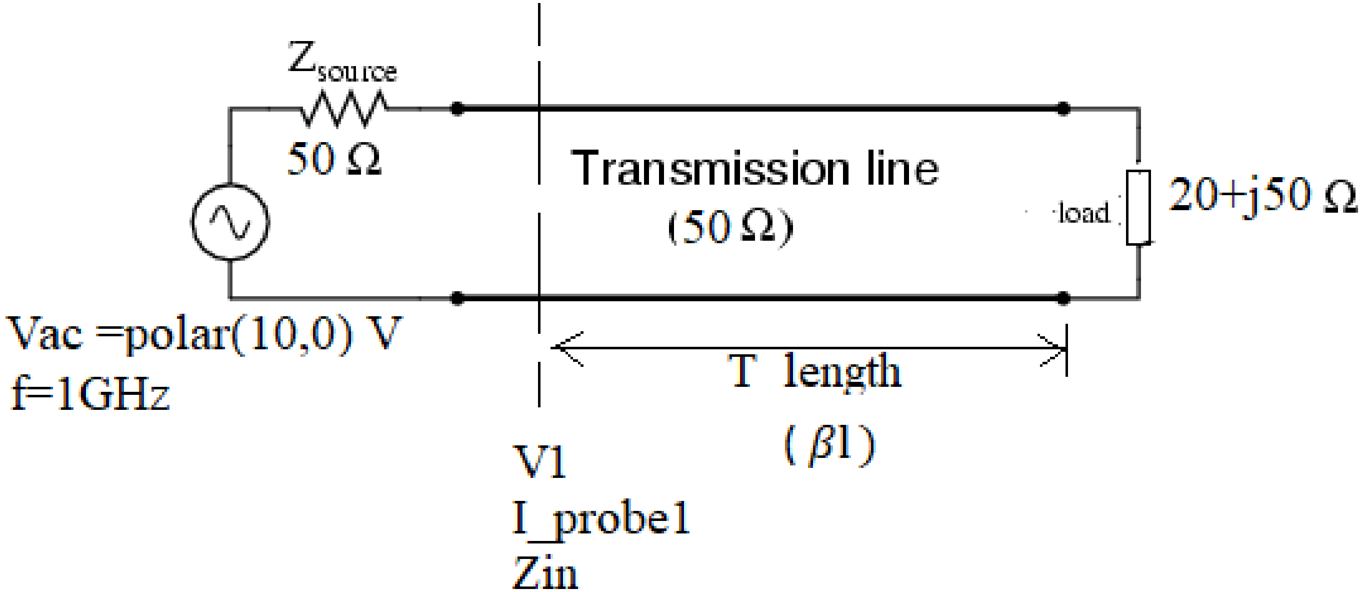
-Compute magnitude of voltage , current at load.

-verify input impedance at load from voltage/current

Equals load impedance.

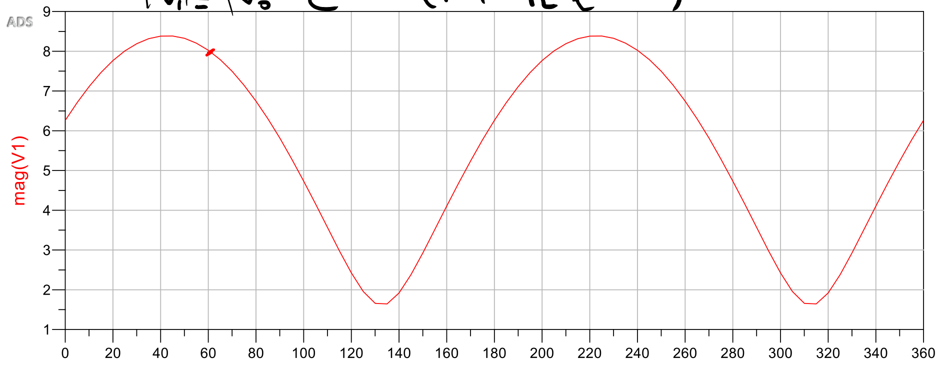
-find max voltage value and its position

-find min voltage value and its position



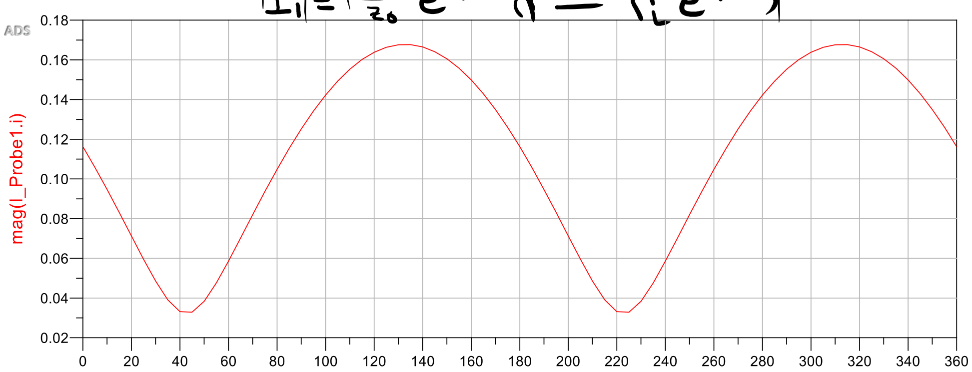
Terminated transmission line repeats its voltage mag.,current mag. and impedance each $\lambda/2$

$$|V_1| = |V_0^+ e^{-j\beta z}(1 + \Gamma_L e^{2j\beta z})|$$

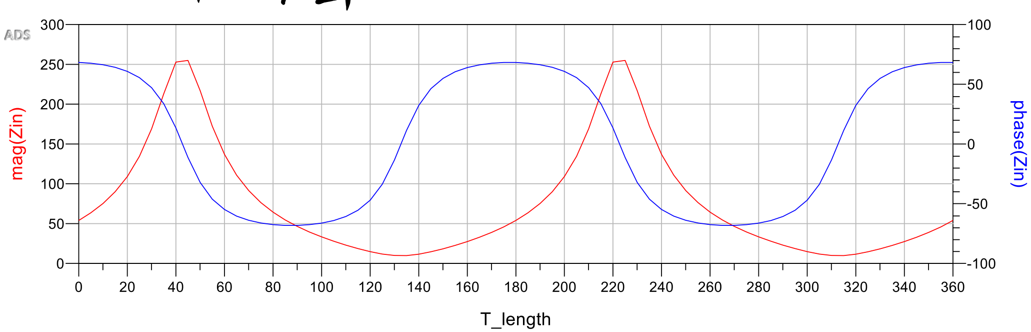


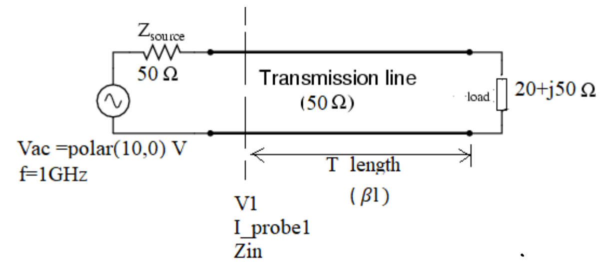
$\uparrow z=0$

$$|I_1| = \left|\frac{V_0^+}{Z_0} e^{-j\beta z}(1 - \Gamma_L e^{2j\beta z})\right|$$



$$Z_1 = V_1 / I_1$$





@ $z=0$ $\quad V_0^+ = 5$ Volt $\quad Z_g = Z_0$

$$V_1 = V_0^+(1 + \Gamma_L), \quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V_1 = 5(1 + 0.678\angle 85.43)$$

$$V_1 = 6.26 \angle 32.67$$

$$I_1 = \frac{V_0^+}{Z_0}(1 - \Gamma_L) = 0.116\angle -35.53$$

$$Z_1 = V_1 / I_1 = 53.9\angle 68.2$$

$$= Z_L$$

$$Z_L = 20 + j50 = 53.85\angle 68.2$$

$$\left.\begin{aligned} V(\ell + \lambda/2) &= -V(\ell) \\ I(\ell + \lambda/2) &= -I(\ell) \end{aligned}\right\} \quad Z_{in}(\ell + \tfrac{\lambda}{2}) = Z_{in}(\ell)$$

$$V(\ell + \lambda) = V(\ell), \quad I(\ell + \lambda) = I(\ell)$$

for $\beta l = 60^\circ$ $\quad V_0^+ = 5 \quad \Gamma_L = 0.678\angle 85.43$

Find $V(\beta l = 60)$, $I(\beta l = 60)$, $Z(\beta l = 60)$

Soln

$$V = V_0^+ e^{j\beta l}\left(1 + \Gamma_L e^{-2j\beta l}\right)$$

$$= 5\angle 60\left(1 + 0.678\angle 85.43 \angle -120\right)$$

$$= 5.56 + j5.79 = 8\angle 46.13^\circ \quad \text{Volt}$$

$$I = \frac{V_0^+}{Z_0} e^{j\beta l}\left(1 - \Gamma_L e^{-2j\beta l}\right)$$

$$= \frac{5}{50}\angle 60\left(1 - 0.678\angle 85.43 \angle -120\right)$$

$$= -0.011 + j0.057 = 0.0586\angle 101$$

$$Z = \frac{V}{I} = \frac{8\angle 46.13}{0.0586\angle 101} = 78.48 - j111.7 = 136.6\angle -54.9$$

$V_0^+ = 5$ Volt $\quad \Gamma_L = 0.678 \angle 85.43$

$$|V| = |V_0^+| \, \left| 1 + |\Gamma_L| e^{j\phi} e^{-2j\beta l} \right|$$

$$|V_{max}| = |V_0^+| (1 + |\Gamma_L|) = 5 \times 1.678 = 8.4 \text{ Volt}$$

$$|V_{min}| = |V_0^+| (1 - |\Gamma_L|) = 5 \times 0.322 = 1.6 \text{ Volt}$$

Max @ $\phi - 2\beta l = 0 \Rightarrow \beta l = \frac{85.43}{2} = 42.7^\circ$

Min @ $\phi - 2\beta l = \pi \Rightarrow \beta l = \frac{85.43 - 180}{2}$

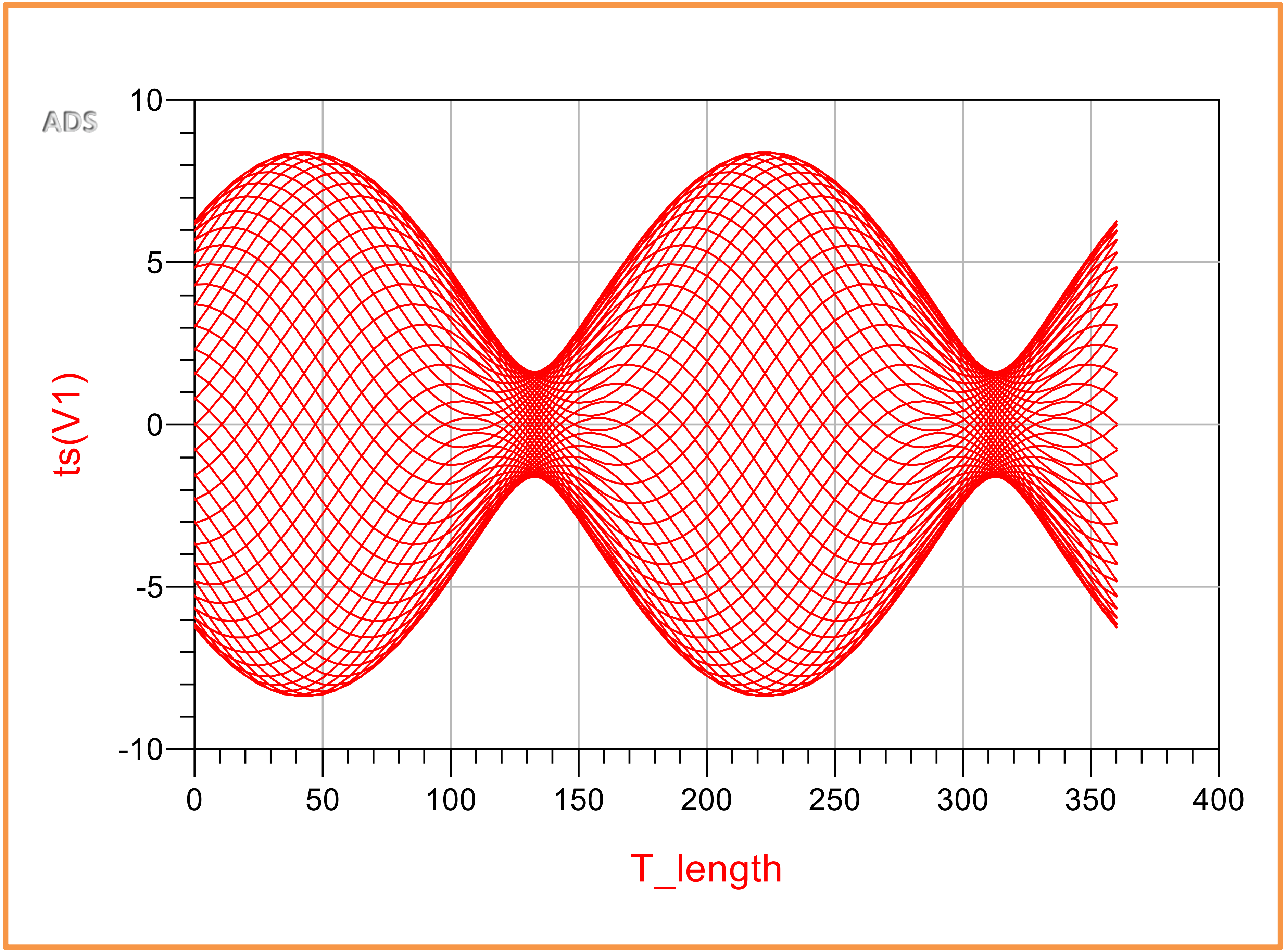
$= -47.28 + 180$

$= 132.7^\circ$

at $l = \lambda/2 \quad \beta l = 180$

magnitude of Voltage Repeats every $l = \frac{\lambda}{2}$

as $|V(z + \lambda/2)| = |V(z)|$ or $\left| 1 + |\Gamma_L| e^{j\phi} e^{-2j\beta l} e^{-2j \times 180} \right| = \left| 1 + |\Gamma_L| e^{j\phi} e^{-2j\beta l} \right|$


ADS
10
5
0
-5
-10
ts(V1)
0
50
100
150
200
250
300
350
400
T_length